EXAMINATIONS: Main Exam - June 2016

SUBJECT, COURSE AND CODE: INTERMEDIATE ECONOMETRICS
ECON7IE P1

DURATION OF EXAM: Three hours  TOTAL MARKS: 100

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Internal Examiner: Dr P Nyatanga

INSTRUCTIONS TO STUDENTS:
This paper consists of THIRTEEN (13) pages in total, including this page. The last five of these pages contain a formula sheet and statistical tables.

1. This exam consists of seven questions.
2. Answer all questions.
3. Please show all workings – part marks may be allocated to correct workings.
4. You may use a scientific calculator.
a) A researcher is interested in analysing the factor which influences the price of property. Using a sample of 506, he obtains the following results:

```
. reg lprice crime rooms lproptax

Source | SS      df       MS              Number of obs =     506
-------------+--------------------------------------
Model | 50.9013457     3  16.9671152           Prob > F      =  0.0000
Residual | 33.6808792   502  .067093385           R-squared     =  0.6018
-------------+--------------------------------------
Total |   84.582225   505  .167489554           Root MSE      =  .25902

------------------------------------------------------------------------------
 lprice |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+------------------------------------------------------------
  crime | -.0133218   .0016121  -8.26   0.000  -.0164891    -.0101545
 rooms |   .2889802   .0172329   16.77   0.000    .2551227    .3228377
 lproptax | -.2608734   .0357288  -7.30   0.000  -.3310697    -.1906777
    _cons |   9.720548   .2562679   37.93   0.000    9.217059    10.22404
------------------------------------------------------------------------------

Where:  lprice   median housing price, $, in log form
        crime   per capita crimes committed
        rooms   avg number of rooms
        lproptax property tax per $1000, in log form
```

(i) Interpret the coefficients \textit{crime}, and \textit{lproptax}, in the estimated function above, and briefly explain whether their signs and significance agree with your expectations. \[6\]

(ii) What other variable could be added to improve this model? Explain \[2\]

(iii) Test the hypothesis that \textit{rooms} has no significant effect on the median housing price, at the 1\% significance levels. \[4\]

(iv) Conduct an overall significance test of the whole model at 5\% significance level. What do you conclude? \[4\]
The output below shows an estimated earnings function for wage-employed male workers, using Labour Force Survey data from 2007:

```
. reg lnhwage coloured indian white exp expsq yrschool col_school ind_school whi_school unionmem metro married semiskill skilled if gender==1 & age>=16 & age<=65 & employee==1

Source | SS df MS
----------+--------------------------------------------------
Model | 5725.83265 14 408.988047
Residual | 5640.32016 10864 .519175272
----------+--------------------------------------------------
Number of obs = 10879
F( 14, 10864) = 787.76 Prob > F = 0.0000
Adj R-squared = 0.5031
R-squared = 0.5038
Total | 11366.1528 10878 1.04487524 Root MSE = .72054

------------------------------------------------------------------------------
| Coef. Std. Err. t P>|t| [95% Conf. Interval]
-------------+--------------------------------------------------------------------------
coloured | .1044542 .0435075 2.40 0.016 .0191715 .1897369
indian | -.3019999 .2272505 -1.33 0.184 -.7474524 .1434525
white | .4389166 .1491008 2.94 0.003 .1466518 .7311814
exp | .0181409 .0021629 8.39 0.000 .0139012 .0223807
expsq | -.0001341 .0000393 -3.41 0.001 -.0002112 -.000057
yrschool | .0790252 .0028091 28.13 0.000 .0735189 .0845316
col_school | .0068471 .0047385 1.44 0.148 -.0024412 .0161354
ind_school | .0535285 .0193948 2.76 0.006 .0155127 .0915442
whi_school | .0254973 .0122095 2.09 0.037 .0015644 .0494302
unionmem | .5303859 .0158319 33.50 0.000 .4993525 .5614192
metro | .1923475 .0185126 10.39 0.000 .1560593 .2286356
married | .2092208 .01694 12.35 0.000 .1760153 .2424263
semiskill | .8147558 .0259046 31.45 0.000 .7639781 .8655335
skilled | .2349638 .0166825 14.08 0.000 .202263 .2676645
_cons | .7690114 .0375135 20.50 0.000 .6954781 .8425447

------------------------------------------------------------------------------
```

where  
lnhwage = natural log of hourly wage 
coloured = 1 if coloured 
indian = 1 if indian 
white = 1 if white 
exp = years of work experience 
expsq = experience squared 
yrschool = years of education completed 
col_school = coloured*yrschool 
ind_school = indian*yrschool 
whi_school = white*yrschool 
unionmem = 1 if member of a union 
metro = 1 if metropolitan area 
marrried = 1 if married 
semiskill = 1 if works in semi-skilled occupation 
skill = 1 if works in skilled occupation

**a)** How does additional work experience affect earnings, *ceteris paribus*? Discuss briefly. [4]

**b)** Roughly sketch, on the same set of axes, the returns to education for African and Indian men, holding all else constant at zero. The graph does not need to be drawn accurately to scale. [4]
We then estimate a second model:

```stataeg lnhwage coloured indian white exp expsq yrschool wsecondary ztertiary unionmem metro married semiskill skilled if gender==1 & age>=16 & age<=65 & employee==1
```

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>Number of obs = 10879</th>
</tr>
</thead>
</table>
| F( 13, 10865) = 883.21
| Model | 5839.93248 | 13 | 449.225576 |
| Residual | 5526.22033 | 10865 | .508625893 |
| Total | 11366.1528 | 10878 | 1.04487524 | Root MSE = .71318 |

| lnhwage | Coef.   | Std. Err. | t    | P>|t|   | [95% Conf. Interval] |
|---------|---------|-----------|------|-------|---------------------|
| coloured | .1789764 | .0179055 | 10.00 | 0.000 | .1438783 to .2140746 |
| indian | .3151145 | .0411092 | 7.67 | 0.000 | .234533 to .3956961 |
| white | .7162793 | .0297095 | 25.66 | 0.000 | .6615717 to .770987 |
| exp | .0224056 | .0021962 | 10.20 | 0.000 | .0181006 to .0267105 |
| expsq | -.0002396 | .0000401 | -5.97 | 0.000 | -.0003182 to -.0001609 |
| yrschool | .0648488 | .0076038 | 8.53 | 0.000 | .0499439 to .0797536 |
| wsecondary | .0380371 | .0046741 | 8.14 | 0.000 | .0288751 to .0471992 |
| ztertiary | .0931454 | .0115669 | 8.05 | 0.000 | .0704722 to .1158185 |
| unionmem | .516583 | .0157052 | 32.89 | 0.000 | .4857981 to .547368 |
| metro | .2015686 | .0183372 | 10.99 | 0.000 | .1656248 to .2375124 |
| married | .2081658 | .0167634 | 12.42 | 0.000 | .1753066 to .241025 |
| semiskill | .2375131 | .01654 | 14.36 | 0.000 | .20509 to .2699363 |
| skilled | .6964834 | .0268105 | 25.98 | 0.000 | .643933 to .7490368 |
| _cons | .9248832 | .0383077 | 24.14 | 0.000 | .849793 to .9999733 |

where

- `wsecondary` = `(yrschool - 7)*(1 if yrschool > 7, 0 otherwise)`
- `ztertiary` = `(yrschool - 12)*(1 if yrschool > 12, 0 otherwise)`

c) Discuss the returns to education. Explain briefly how these differ from the previous model. [4]

d) Test the hypothesis that living in a metropolitan area increases earnings by 25%, *ceteris paribus*, using the second model. Show all workings. [4]

**Question 3** [12 marks]

a) Explain briefly, in your own words, what autocorrelation is, and under what circumstances it might occur. [4]

b) We estimate an aggregate savings function, using US data collected for 84 quarters from 1990 to 1996.

   (i) We then conduct a Breusch-Godfrey test, using four lags of the residuals, and obtain a chi-squared test statistic of 16.2. Write out the null hypothesis, and conduct the test at a 1% significance level. [4]

   (ii) Based on the test results above, do you need to be concerned about your model? Why or why not? Explain briefly how you would proceed. [4]
Consider the following demand-and-supply model for loans of commercial banks to businesses:

**Demand:** 
\[ Q^d_t = \alpha_1 + \alpha_2 R_t + \alpha_3 RD_t + \alpha_4 IPI_t + u_{1t} \]

**Supply:** 
\[ Q^s_t = \beta_1 + \beta_2 R_t + \beta_3 RS_t + \beta_4 TBD_t + u_{2t} \]

Where \( Q \) = total commercial bank loans ($billion); 
\( R \) = average prime rate; \( RS \) = 3-month Treasury bill rate; 
\( RD \) = AAA corporate bond rate; \( IPI \) = Index of Industrial Production; and 
\( TBD \) = total bank deposits.

a) Are the demand and supply functions identified? Show all workings. [4]

b) Derive an expression for the reduced form equation for \( R_t \). [4]

c) How would you go about estimating the demand and supply functions listed above? Do not use any Stata commands in your answer. [4]
A logit model is used to estimate the probability of cheating, using a sample of 601 married individuals. The following estimation results are obtained:

\[
\text{. logit affair educ age male yrsmarr kids relig ratemarr}
\]

Iteration 0:  log likelihood = -337.68849  
Iteration 1:  log likelihood = -305.98788  
Iteration 2:  log likelihood = -304.85259  
Iteration 3:  log likelihood = -304.84818  
Iteration 4:  log likelihood = -304.84818

Logistic regression                        Number of obs   =        601  
                                         LR chi2(7)      =      65.68  
                                         Prob > chi2     =     0.0000  
Log likelihood                          -304.84818  
Pseudo R2                           0.0973

|          | Coef.   | Std. Err. | z     | P>|z|    | [95% Conf. Interval] |
|----------|---------|-----------|-------|--------|---------------------|
| educ     | .0305846| .0454078  | 0.67  | 0.501  | -.058413            |
| age      | -.0437373| .0181916  | -2.40 | 0.016  | -0.0793922          |
| male     | .3164296| .2243866  | 1.41  | 0.158  | -.0080824           |
| yrsmarr  | .0952295| .0321745  | 2.96  | 0.003  | .0321687            |
| kids     | .3791557| .0321745  | 2.96  | 0.003  | .0321687            |
| relig    | -.3254421| .0897371  | -3.63 | 0.000  | -.501326            |
| ratemarr | -.4700074| .0321745  | -5.18 | 0.000  | -.6479596           |
| _cons    | 1.336704| .8818964  | 1.52  | 0.130  | -.3917809           |

Where:  
affair = 1 if he/she had at least one affair, and 0 otherwise  
educ = years schooling  
age = in years  
male = 1 if male, and 0 otherwise  
yrsmarr = years married  
kids = 1 if one has kids, and 0 otherwise  
raterel = how one rates his/her level of religion - 5 = very religious,  
           1 = not religious  
ratehap = how one rates his/her level of happiness - 5 = very happily married,  
           1 = very unhappy

a) What is the effect of, yrsmarr, kids, and ratehap on the odds of cheating? Briefly offer a plausible explanation to these effects.  

b) Predict the probability that a woman with the following characteristics will have an affair (cheat)

educ = 12  
yrsmarr = 3  
raterel = 1  
ratehap = 1
c) How will the probability of having an affair change should a woman with the same characteristics as in b) becomes very religious? [3]

d) The model is re-estimated, excluding the variable two variables and produces a log likelihood of -326.9329. Conduct a hypothesis test comparing the full and the restricted models at a 1% level of significance. Write out the null and alternative hypotheses and show all working. Which model do you prefer? [4]

**Question 6** [16 marks]

Consider the data on US unemployment for 1948 to 1996, illustrated in the figure below:

![Unemployment Rate Time Series](image)

a) Explain what is meant by the cyclical component of a time series. Do you see any evidence of a cyclical component to the unemployment rate time series? Explain. [3]

b) For the following regression, the sample size is 48, $U_t$ is the unemployment rate at time $t$ and estimated standard errors are presented in brackets:

$$
\Delta U_t = 1.630 \cdot U_{t-1} + 0.016 t \\
(0.576) (0.117) (0.013)
$$

Calculate the test statistic and use it to conduct a unit root test for stationarity at a 5% significance level. Show all working. [4]
c) Using your answer from b) and the Stata output below, how would you describe the nature of the unemployment time series? Explain. [3]

```
. dfuller D.unemployment

Dickey-Fuller test for unit root Number of obs = 47

---------- Interpolated Dickey-Fuller ----------
Test 1% Critical 5% Critical 10% Critical
Statistic  Value  Value  Value
------------------------------------------------------------------------------
Z(t) -6.921 -3.600 -2.938 -2.604
MacKinnon approximate p-value for Z(t) = 0.0000
```

d) You discover that there is a cointegrating relationship between inflation and unemployment.
(i) Write, in general terms (i.e. using symbols) an equation for the error correction model that you could estimate for relationship between inflation and unemployment. [2]

(ii) What sign would you expect on the error correction terms, and why? [2]

(iii) Explain what it means if the coefficient on the error correction is large. [2]

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**Question 7** [10 marks]

a) As part of its research, a consumer organisation collect information from companies about their advertising budgets. It suspects that the advertising data thus collected are not entirely accurate. Briefly discuss the consequences for any econometric analysis if:
(i) Inaccurate advertising data are used as an explanatory variable in a regression. [3]

(ii) Inaccurate advertising data are used as a dependent variable in a regression. [3]

b) Which of the following two types of model misspecification has more serious consequences: omitting a relevant variable or including an irrelevant variable? Explain your answer in detail. [4]

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**Total: 100 marks**
**Formula Sheet**

**Test statistics:**

\[ t = \frac{b - \beta^*}{\text{se}(b)} \]

\[ F = \frac{R^2/(k-1)}{(1-R^2)/(n-k)} \]

\[ F = (\text{RSS}_R - \text{RSS}_U)/k \sim F_{k,n-2k} \]

\[ LR = 2\ell_n L_1 - 2\ell_n L_0 \]

\[ DF = \frac{\hat{\theta} - 1}{\text{se}(\hat{\theta})} \]

**Expected value of coefficient on \( X \):**

\[ E(b) = \beta + \frac{\text{cov}(X,u)}{\text{var}(X)} \]

**Measurement error in \( X_2 \):**

\[ E(b_2) = \beta_2 + \frac{\text{var}(X_2)}{\text{var}(X_2) + \text{var}(\nu)} \]

**Variable \( X_3 \) omitted from the model:**

\[ E(b_2) = \beta_2 + \beta_3 \frac{\text{cov}(X_2,X_3)}{\text{var}(X_2)} \]

**Logistic distribution:**

\[ \text{Prob}(Y = 1) = \text{logistic}(\beta_1 + \beta_2 X_2 + ...) = \frac{e^{\beta_1 + \beta_2 X_2 + ...}}{1 + e^{\beta_1 + \beta_2 X_2 + ...}} \]

**Normal distribution:**

\[ \text{Prob}(Y = 1) = \text{normal}(\beta_1 + \beta_2 X_2 + ...) = \int_{-\infty}^{\beta_1 + \beta_2 X_2 + ...} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt \]

**Percentage change in odds = \( 100(e^{\beta_k} - 1) \)**

**Variance of the error terms:**

\[ \hat{\sigma}^2_{\text{ols}} = \frac{\text{SSE}}{n} \]

\[ \hat{\sigma}^2_{\text{ml}} = \frac{\text{SSE}}{n - k} \]

**Model fit:**

\[ LR = \text{pseudo} R^2 = 1 - \frac{\ell_n L_1}{\ell_n L_0} \]
### Critical Values for Dickey-Fuller unit root tests

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#### TABLE A-2

**PERCENTAGE POINTS OF THE \( t \) DISTRIBUTION.**

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Note: The smaller probability shown at the head of each column is the area in one tail; the larger probability is the area in both tails.

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### Table A-1b: Cumulative Probabilities of the Standard Normal Distribution

| Entry is area A under the standard normal curve from $-\infty$ to $Z(A)$ |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $z$             | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
| .0               | .5900 | .5640 | .5380 | .5120 | .4860 | .4600 | .4340 | .4080 | .3820 | .3560 |
| .1               | .6868 | .6604 | .6343 | .6082 | .5821 | .5560 | .5299 | .5038 | .4778 | .4517 |
| .2               | .7922 | .7653 | .7386 | .7117 | .6848 | .6580 | .6312 | .6045 | .5779 | .5514 |
| .3               | .8920 | .8651 | .8382 | .8112 | .7841 | .7570 | .7300 | .7030 | .6759 | .6490 |
| .4               | .9683 | .9420 | .9156 | .8891 | .8629 | .8370 | .8117 | .7861 | .7597 | .7331 |
| .5               | .9909 | .9772 | .9631 | .9491 | .9354 | .9217 | .9080 | .8945 | .8810 | .8675 |
| .6               | .9973 | .9998 | .9950 | .9900 | .9850 | .9797 | .9740 | .9679 | .9614 | .9543 |
| .7               | .9995 | .9999 | .9997 | .9996 | .9995 | .9994 | .9993 | .9991 | .9989 | .9986 |
| .8               | 1.0000| 1.0000| 1.0000| 1.0000| 1.0000| 1.0000| 1.0000| 1.0000| 1.0000| 1.0000|
TABLE A.4  UPPER PERCENTAGE POINTS OF THE χ² DISTRIBUTION.

Example

\[ \Pr(\chi^2 > 10.85) = 0.05 \]
\[ \Pr(\chi^2 > 23.63) = 0.05 \text{ for } df = 20 \]
\[ \Pr(\chi^2 > 31.31) = 0.00 \]

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*For df greater than 100 the expression \( \frac{X^2}{n} \) = \( \chi^2 \) characterizes the standardized normal distribution, where \( n \) represents the degrees of freedom.