EXAMINATION: JUNE 2016

SUBJECT, COURSE AND CODE:
ECONOMICS HONOURS: MICROECONOMICS: ECON7MI

DURATION: THREE HOURS

TOTAL MARKS: 150

External Examiner: Dr Marisa von Fintel
Internal Examiner: Dr G. Bokana
Dr B. Rhodes
Dr J. Mbonigaba

INSTRUCTIONS TO CANDIDATES:

1. You may use calculators
2. You should answer all the questions from Part A and choose ONE question from each section (1 to 3) in Part B.
3. You may keep your exam question paper.
4. Please answer EACH question in a separate answer book
5. If you have answered all the necessary questions, you should hand in at least 6 answer books.
Part A

[75 marks]

Answer all three questions in Part A.
Answer each question in a separate answer book

Question 1

A firm has two variables factors ($x_1$ and $x_2$) and a production function:

$$f(x_1, x_2) = x_1^{1/2}x_2^{1/4}.$$ 

The price of its output is 4. Factor 1 receives a wage of $w_1$ and factor 2 receives a wage of $w_2$.

a) Write an equation that says that the value of the marginal product of factor 1 is equal to the wage of factor 1. [4]

b) Write an equation that says that the value of the marginal product of factor 2 is equal to the wage of factor 2. [4]

c) Calculate the amounts of factors 1 and 2 that maximize the firm’s profits as a function of $w_1$ and $w_2$. [10]

d) If the wage of factor 1 is 2, and the wage of factor 2 is 1, calculate how many units of factors 1 and 2 will the firm demand? [4]

e) Calculate how much output will it produce? [3]

f) Calculate how much profit will it make? [2]
Question 2

A consumer chooses weekly quantities of X and Y in order to maximize the utility function:

\[ U = X^{0.6} Y^{0.6} \]

a) The utility function above can be written in general form as \( U = X^\alpha Y^\beta \). Use a Lagrangian multiplier to derive the general form of the demand function for X and Y specified by this general form utility function. 

[9]

b) Use the demand functions obtained in a) to determine how much X and Y the consumer will actually purchase given her total income is R4100 and the prices of X and Y are R10 and R4 respectively.

[2]

c) Use your answers to part b) to calculate the marginal rate of substitution between the goods.

[4]

d) Calculate and comment on the budget shares of each good at equilibrium.

[4]

e) Sketch the income expansion path and the Engel curve for good X. What type of preferences are described by such diagrams?

[6]

[25 marks]
Question 3

a) Suppose that a consumer has the following utility function: \( U(x_1, x_2) = x_1^{1/3} x_2^{2/3} \)
She originally faces prices (1,1) and has an income of 200. Then the price of \( x_1 \) increases to R2.

i) Determine the demand functions (no lengthy calculations needed here) (2)

ii) Estimate the value of the compensating variation for this consumer. (6)

iii) Estimate the value of the equivalent variation for this consumer. (6)

iv) Interpret the meaning of this values obtained in a) and b) above (2)

b) A producer of shoes has the following supply function \( P = 10q + 500 \) and the market price is R600. If the market price increases from R600 to R750

i) Estimate the change in producer surplus using definite integrals (6)

ii) Interpret the meaning of the value you get (3)

[25 marks]
Part B  [75 marks]

Answer ONE question from each section (1 to 3).
Answer each question in a separate answer book

Section 1 (answer ONE question)

Question 1

In Durban, a duopoly market has two bakers, Firm 1 and Firm 2. Firm 1 has constant marginal costs of R1 per loaf of bread. Firm 2 has constant marginal costs of R2 per loaf. Fixed costs are zero for both firms. The inverse demand function for bread in Durban is \( P(y) = 6 - 0.01y \), where \( y \) is the total number of loaves sold per day and \( y = y_1 + y_2 \).

a) What is the Cournot equilibrium quantity for each firm in this market?  [14]

b) What is the Cournot equilibrium price in this market?  [6]

c) What is the profit for each firm in this market?  [5]

[25 marks]

OR

Question 2

A firm faces a production function of \( Q = 150\sqrt{LK} \). This firm has orders for 1500 units of output. The price of labour is R15 per unit and the price of capital is R15 per unit.

a) Set up the Lagrangian function for the firm’s cost minimisation problem, denoting the Lagrange multiplier by \( \lambda \).  [4]

b) Find the first-order conditions for the choice of \( L \), \( K \) and \( \lambda \).  [8]

c) Find the cost-minimizing combination of inputs - \( L \), \( K \) and \( \lambda \).  [6]

d) Derive an expression for the firm’s technical rate of substitution (TRS). Briefly explain what the TRS calculates.  [4]

e) Calculate the elasticity of substitution for this firm. What type of production function is this?  [3]

[25 marks]
Section 2 (answer ONE question)
Remember to start a new answer book for section 2

Question 3

a) Suppose that the value of fish stocked in a particular lake at time $t$ is given by $F(t)$. Explain why the best time to harvest the fish is when $r = g$, where $r$ is the rate of interest and $g$ is the rate of growth of the fish stock, i.e.

$$g = \frac{F'(t)}{F(t)} = \frac{1}{F(t)} \frac{dF(t)}{dt}.$$  

n.b a second order condition is not required.  

b) Consider the following returns from investments where warm and cold summers are equally likely to happen. You have $2000 to invest.

<table>
<thead>
<tr>
<th></th>
<th>Current share price</th>
<th>Value if warm summer</th>
<th>Value if cold summer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ice cream Company</td>
<td>$50</td>
<td>$100</td>
<td>$200</td>
</tr>
<tr>
<td>Hot soup Company</td>
<td>$50</td>
<td>$200</td>
<td>$100</td>
</tr>
</tbody>
</table>

i) What is the expected value and variance of your portfolio if you invest all $2000 in the Hot Soup Company?

[4]

ii) What is the expected value and variance of your portfolio if you invest all $1500 in the Hot Soup Company and $500 in the Ice Cream company?

[6]

iii) Briefly compare the risk profile of the investment portfolios found in (i) and by calculation, show how risk can be reduced to zero.

[5]

[25 marks]
Question 4

An isolated village produces potatoes and does not trade with the outside world. Good
harvests alternate with bad harvests. This year the harvest will be 5000kgs. Next year
it will be 1200kgs. Potatoes can be stored, but rats will eat 20% of what is stored in a
year. The villagers have the Cobb-Douglas utility function $U(c_1, c_2) = c_1c_2$, where $c_1$
is consumption this year and $c_2$ is consumption next year.

a) Draw a budget line for the village with this year’s consumption on the
horizontal axis and next year’s consumption on the vertical axis. On your
graph show the formula and quantities at which the budget line intercepts
the vertical and horizontal axes. Label the endowment amounts.

b) How much will the villagers consume this year?

c) How much corn will the rats eat?

d) How much corn will the villagers eat next year?

e) Show that future consumption equals the future stock of corn. In general
terms, briefly comment on the implicit price of the first period relative to
the second.

f) Show the discounted version of your answer to part e). In general terms
briefly comment on the implicit price of the second period relative to the
first.

[25 marks]
Section 3 (answer ONE question)
Remember to start a new answer book for section 3

Question 5

a) Ann has 10 units of good x and 3 units of good y. Bob has 4 units of good x and 8 units of good y. At these endowments, the marginal rates of substitution (of good x and good y) of Ann and Bob are respectively $MRS_A = 4$ and $MRS_B = 2$

i) Represent the allocations mentioned above in an Edgeworth box
(6)

ii) Is this allocation Pareto efficient? Explain
(4)

b) Consider the following gross demand functions and initial endowments in the context of pure exchange general equilibrium analysis.

$x_1^A(p_1, p_2)$ is the gross demand for good 1 by agent A

$x_1^B(p_1, p_2)$ is the gross demand for good 1 by agent B

$x_2^A(p_1, p_2)$ the gross demand for good 2 by agent A

$x_2^B(p_1, p_2)$ the gross demand for good 2 by agent B

$\sigma_i^A$ is the initial endowment in good 1 to A

$\sigma_i^B$ is the initial endowment in good 1 to B

$\sigma_i^A$ is the endowment in good 2 to A

$\sigma_i^B$ is the endowment in good 2 to B

Prices of Good 1 and Good 2 that result in the general equilibrium are $p_1^*$ and $p_2^*$ respectively.

i) Using the gross demands for good 1 and good determine the general Equilibrium equations
(6)

ii) Using the net demands for good 1 and good 2, determine the general equilibrium equations
(6)

iii) What does each of the equation i a) and ii) mean?
(3)

[25 marks]
OR

QUESTION 6

Each entry-level software programmer in Palo Alto, California, has either high or low ability. All potential employers value a high-ability worker at $12,000 per month and a low-ability worker at $6,000.

The supply of high-ability workers is \( Q_H^S = 0.05(W - 2000) \)
and the supply of low-ability workers is \( Q_L^S = 0.1(W - 2000) \),
where \( W \) is the monthly wage.

a) If workers' abilities are observable to employers, what are the equilibrium wages? How many workers of each type do employers hire? (6)

b) If workers' abilities are not observed by employers, what is the equilibrium wage? (8)

c) How many workers of each type do employers hire? (5)

d) Explain the inefficiencies arising from the asymmetric information in this market (6)

[25 marks]