UNIVERSITY OF KWAZULU-NATAL
SCHOOL OF CIVIL ENGINEERING, SURVEYING AND CONSTRUCTION
PROGRAMME OF SURVEYING AND MAPPING

DECEMBER 2016 EXAMINATIONS

Subject, Course and Code:

COORDINATE SYSTEMS AND GEODETIC PROJECTIONS

ENSV3CG

DURATION: 3 HOURS

AVAILABLE MARKS: 100
TOTAL MARKS: 100

Internal Examiner: Mr J Jackson
External Examiner: Mr A M Forbes

Instructions to candidates:
Answer any FIVE questions.
Question 1: Geographic and Astronomical co-ordinate systems.

(a) When we talk about the ITRF and the ITRS:
   (i) Which of these is called "conventional" and what does the word "conventional" mean in this context? (2)
   (ii) If your smartphone gives you a geographical position on Google Maps is that in the ITRS or ITRF? Say why. (2)
   (iii) If you measure a polar using a total station, what extra information do you need to add to be able to find its position in the ITRF? (2)

(b) Suppose you want keep the rounding errors from keyed-in geographical positions to less than 0.5mm on the ground:
   (i) To how many decimals of a decimal degree do you need to specify a latitude? (2)
   (ii) To how many decimals of an arc second do you need to specify a latitude? (2)
   (iii) Are your answers to (i) and (ii) above also good enough for the decimals needed when keying in a longitude? Explain your answer. (3)

(c) What is the name of the origin of Right Ascension and why does it move with time? (3)

(d) Using the table below, calculate the ITRF coordinates of station SUTH on the day of this exam.

<table>
<thead>
<tr>
<th>STATION</th>
<th>X/(\text{m})</th>
<th>Y/(\text{m})</th>
<th>Z/(\text{m})</th>
<th>Sigm(\text{a})</th>
<th>D(\text{OLN})</th>
<th>D(\text{ATA}_\text{START})</th>
<th>D(\text{ATA}_\text{END})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hartebeespoort</td>
<td>5085657.257</td>
<td>2670325.112</td>
<td>-2768481.145</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Pretoria</td>
<td>5088223.083</td>
<td>2719127.014</td>
<td>-2756392.621</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Sutherland</td>
<td>5041274.339</td>
<td>1916054.150</td>
<td>-3397078.939</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>

(4)

[20]
Question 2: Spherical trigonometry.

(a) To which two great cultures do we owe the ideas that (i) the positions of features on the spherical Earth can be measured and published (ii) the sphere can be used as a computing surface? Give a date and the name of a pioneer of each idea. (4)

(b) Calculate the shortest distance from Addis Ababa (9°N 38.8°E) to Durban (29.9°S 31.0°E) in Nautical Miles and the starting bearing in decimal degrees. (8)

(c) Suppose you plan to place two waypoints equally spaced along the Great circle route from Addis Ababa to Durban. Calculate the geographical coordinates of the first waypoint (8) [20]

Question 3: The ellipsoidal reference system.

(a) The Wikipedia entry on the UTM defines the third flattening as $n=f/(2-f)$ where $n$ is the flattening. Show that this is the same as the definition given in the formula sheet at the end of this paper. (4)

(b) For the position of Addis Ababa (9.0°N 38.8°E) on the WGS84 ellipsoid, calculate
   (i) The radius of curvature at bearing 0 degrees (2)
   (ii) The radius of curvature at bearing 90 degrees (2)

(c) Suppose a point has geocentric coordinates
   $(X=4911736.491m,Y=3949137.529m,Z=991529.661m)$
   (i) In which hemispheres does this point lie? (2)
   (ii) Calculate its longitude. (3)
   (iii) Carry out ONE iteration to calculate its latitude and ellipsoidal height (7) [20]
Question 4: Cartesian co-ordinate systems

(a) The following observations were taken from a total station setup at P with H1=1.54m

<table>
<thead>
<tr>
<th>Target</th>
<th>Hz</th>
<th>Va</th>
<th>Sslope</th>
<th>Hsig</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>349°10’13”</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>143°10’20”</td>
<td>88°22’17”</td>
<td>213.225m</td>
<td>1.32m</td>
</tr>
</tbody>
</table>

(i) Explain why Curvature correction should not be applied to the LAS height difference in the baseline LAS baseline P to A, but refraction correction should be.  

(3)

(ii) If N lay exactly North of P, calculate the baseline P to A in a Left handed LAS oriented with the +X axis running Northwards.

(4)

(iii) If P was at position (9.0°N 38.8°E), and neglecting the effects of deflection of the vertical, write down the formula you would use to transform the baseline P to A into the ITRS. Do not do the calculation, just write out the equation.

(5)

(b) In what kinds of survey projects using a total station would you need to take account of deflections of the vertical? Describe two methods that could be used to measure those deflections.

(5)

(c) What measurements for the Cape datum took account of deflection of the vertical?  

(3)

[20]
Question 5 Geodetic projections.
(a) Texas uses five State Plane Systems shown below:

<table>
<thead>
<tr>
<th>Zone Codes (Texas zones are Lambert Conformal Conic projections)</th>
<th>Standard Parallels DD.MM</th>
<th>Origin Long/lat DD.MM</th>
<th>False Easting Northing meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>North TX N 4201</td>
<td>34:39</td>
<td>101:30</td>
<td>200,000</td>
</tr>
<tr>
<td>North Central TX NC 4202</td>
<td>32:08</td>
<td>98:30</td>
<td>600,000</td>
</tr>
<tr>
<td>Central TX C 4203</td>
<td>30:07</td>
<td>100:20</td>
<td>700,000</td>
</tr>
<tr>
<td>South Central TX SC 4204</td>
<td>28:23</td>
<td>99:00</td>
<td>600,000</td>
</tr>
<tr>
<td>South TX S 4205</td>
<td>26:10</td>
<td>98:30</td>
<td>300,000</td>
</tr>
</tbody>
</table>

(i) What is different about the way Texans handle the boundaries between zones or map panels, from South African method? Which method do you think is better and why? (3)

(ii) Why do these zones have “standard parallels”? What is the equivalent of a “standard Parallel” on the UTM and on the SACRS? (4)

(iii) Why do these zones have False Eastings and Northings? (2)

(b) Suppose you have a survey project near Addis Ababa (9.0°N 38.8°E, H=2356m) and you needed to present your results on the UTM

(i) What Zone would your project fall in and what will be its Central Meridian? (3)

(ii) Calculate the approximate Gauss Conform coordinates for the same CM taking X as the arc of meridian from the equator and Y as the small circle distance from the CM. Assume a spherical Earth (4)

(iii) Calculate the combined sea level and scale enlargement correction you expect when using the UTM in your survey area (4)

[20]
Question 6: Plane transformations.

(a) Describe how it is simpler and more accurate to use plane transformations for a datum transformation e.g. Cape to Hart94, than a 3 or 7 parameter Geocentric transformation. (4)

(b) If it is simpler and more accurate to use plane transformations for a datum transformation, why does the International Hydrographic Office publish parameters for geocentric transformations? (3)

(c) Draw a sketch showing the meaning of \(x, y, X, Y, \Delta X, \Delta Y\) and \(\theta\) in the transformation formula:

\[
\begin{pmatrix}
X \\
Y
\end{pmatrix} = \begin{pmatrix} \Delta X \\
\Delta Y
\end{pmatrix} + \mu \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix} \begin{pmatrix} x \\
y
\end{pmatrix}
\] (4)

(d) If the above equation is written as \(X = \Delta X + \mu R_x\), write down the inverse formula for finding \(x\). (4)

(e) What is the minimum number of common points \((x_i, y_i)\) and \((X_i, Y_i)\) needed to solve for the parameters in a plane similarity transformation? (2)

(f) What is the advantage of doing a datum transformation using an affine transformation based on a TIN instead of a Helmert Transformation? (3) [20]
Formula sheet

\[
\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}
\]

\[
\cos a = \cos b \cos c + \sin b \sin c \cos A...
\]

Mercator Projection for Lat, Long in radians:

\[
X = R \log_e \tan(\frac{\pi}{4} + \frac{\phi}{2})
\]

\[
Y = R\lambda
\]

WGS84 ellipsoid a=6 378 137m \hspace{1cm} f=1/298.257223563

Flattening \(f = \frac{a-b}{a}\), 3rd flattening \(n = \frac{a-b}{a+b}\)

\[
e^2 = \frac{a^2 - b^2}{a^2}
\]

\[
\tan \beta = \frac{b}{a} \tan \phi.
\]

\[
W = \sqrt{1 - e^2 \sin^2 \phi}, \hspace{1cm} M = \frac{a(1-e^2)}{W^3}, \hspace{1cm} N = \frac{a}{W}, \hspace{1cm} \frac{1}{R_a} = \frac{\cos^2 \alpha}{M} + \frac{\sin^2 \alpha}{N}
\]

\[
p = \sqrt{x^2 + y^2}, \hspace{1cm} N = \frac{a}{W}; \hspace{1cm} \phi = \tan^{-1} \left( \frac{z}{p \left(1 - e^2 \frac{N}{N+h} \right)} \right); \hspace{1cm} h = \frac{p}{\cos \phi} - N
\]

\[
x = \begin{bmatrix} p \cos \lambda = (N+h) \cos \phi \cos \lambda \\ p \sin \lambda = (N+h) \cos \phi \sin \lambda \\ \{N(1-e^2) + h\} \sin \phi \end{bmatrix}
\]

\[
\begin{bmatrix} x \\ y \\ z \end{bmatrix} = s \begin{bmatrix} \sin \zeta \cos \alpha \\ \sin \zeta \sin \alpha \\ \cos \zeta \end{bmatrix}
\]
\[
\begin{align*}
X' &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix} x \\
Y &= \begin{bmatrix} \cos \beta & 0 & -\sin \beta \\ 0 & 1 & 0 \\ \sin \beta & 0 & \cos \beta \end{bmatrix} y \\
Z &= \begin{bmatrix} x \\ y \\ z \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
X &= \begin{bmatrix} \cos \gamma & \sin \gamma & 0 \\ -\sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} x \\
Y &= \begin{bmatrix} x \\ y \\ z \end{bmatrix}
\end{align*}
\]

\[
x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\eta \\ 0 & \eta & 1 \end{bmatrix} u.
\]

\[
S = H \left( 1 - \frac{h}{R} + \frac{\eta^2}{2R^2} \right).
\]

UTM false scale factor = 0.9996

\[
Zone = 1 + \text{IntegerPartOf} \left( \frac{\lambda^0 + 180^0}{6} \right)
\]

CM' = Zone x 6 - 183°

To find UTM grid values from Gauss conform (Y,X):

<table>
<thead>
<tr>
<th>Southern Hemisphere</th>
<th>Northern Hemisphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>E = 500000 - Y * 0.9996</td>
<td>E = 500000 + Y * 0.9996</td>
</tr>
<tr>
<td>N = 10000000 - X * 0.9996</td>
<td>N = X * 0.9996</td>
</tr>
</tbody>
</table>

To find Gauss Conform coordinates:

<table>
<thead>
<tr>
<th>Southern Hemisphere</th>
<th>Northern Hemisphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y = (500000 - E) / 0.9996</td>
<td>Y = (E - 500000) / 0.9996</td>
</tr>
<tr>
<td>X = (10000000 - N) / 0.9996</td>
<td>X = N / 0.9996</td>
</tr>
</tbody>
</table>